Globe Magazine

A math problem stumped experts for 50 years. This grad student from Maine solved it in days

“That’s ridiculous,” Lisa Piccirillo thought when she first learned about the Conway knot problem. “We should be able to do that.”

By John Wolfson Updated August 20, 2020, 12:32 p.m.

Mathematician Lisa Piccirillo at MIT, where she is now an assistant professor.Webb Chappell

Half a century ago, a brilliant young mathematician named John Horton Conway discovered, of all things, a knot. This knot wasn’t something you’d be likely to encounter in the real world. You could certainly create it out of string if you wanted to, but, generally speaking, it existed only in Conway’s calculations. There are thousands upon thousands of these kinds of conceptual tangles in a bewildering corner of mathematics known as knot theory, but even there Conway’s discovery was special — not so much for what it was, but for what it might or might not be. Yes, that is confusing, but when talking knot theory, it's best to accept that things are going to get a little fuzzy.

In any case, the Conway knot is hardly remarkable at first glance. With just 11 crossings, or places where it overlaps itself, it's rather nondescript by the standards of higher-dimensional knot theory. But the knot has one property that made it the subject of intense mathematical scrutiny. Conway, who died recently at age 82 of complications from COVID-19, made innumerable contributions to the field of mathematics, yet it was his knot that specialists would return to again and again. And again and again, these decorated mathematicians were unable to find a solution to what became known as the Conway knot problem.

The problem had to do with proving whether the Conway knot was something called "slice," an important concept in knot theory that we'll get to a little later. Of all the many thousands of knots with 12 or fewer crossings, mathematicians had been able to determine the sliceness of all but one: the Conway knot. For more than 50 years, the knot stubbornly resisted every attempt to untangle its secret, along the way achieving a kind of mythical status. A sculpture of it even adorns a gate at the University of Cambridge's Isaac Newton Institute for Mathematical Sciences.

Then, two years ago, a little-known graduate student named Lisa Piccirillo, who grew up in Maine, learned about the knot problem while attending a math conference. A speaker mentioned the Conway knot during a discussion about the challenges of studying knot theory. "For example," the speaker said, "we still don't know whether this 11-crossing knot is slice."

That's ridiculous, Piccirillo thought while she listened. *This is 2018. We should be able to do that*. A week later, she produced a proof that stunned the math world.

Knot theory is a sub-specialty of a field of mathematics known as topology, which is concerned with the study of spaces. What's it used for? "The answer one memorizes is that topology is useful for understanding DNA and protein folding," Piccirillo tells me in May as we sit — wearing masks and maintaining a good 10 feet of distance — in an outdoor courtyard not far from where she lives in Harvard Square. "Apparently these things are very long and they like to stick to themselves, so they get all knotted up."

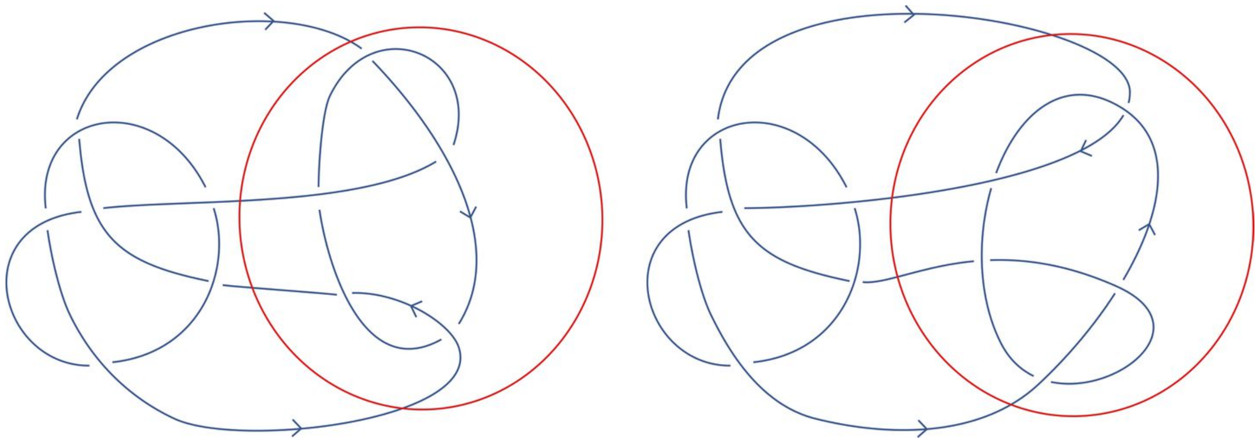
When topologists think of knots, however, they don't imagine a length of rope with a gnarled twist in the middle. To them, a knot is more like an extension cord in which the two ends have been plugged together and the whole thing has been tossed onto the floor in a mess of crisscrosses. It's essentially a closed loop with various places where the loop crosses over itself.

Now let's take one of these knots and think for a moment about the space in which it exists. That space has a fourth dimension, such as time, and to a topologist, our knot is a kind of sphere that sits within it. Topologists see spheres everywhere, but in a specialized way: A circle is a one-dimensional sphere, while the skin surrounding an orange is a two-dimensional sphere. And here is where minds tend to get blown: If we were to take that whole orange and glue it to another one, topologists would see the resulting object as a three-dimensional sphere, one that could be viewed as the skin of a four-dimensional orange. Don't worry if you are unable to conjure such a higher-dimension image for yourself. There are only a couple hundred specialists doing this work in the world, and not even all of them can.

Piccirillo, who graduated from Boston College in 2013, was already well on her way to joining the ranks of those specialists when, in the summer of 2018, the speaker at the math conference said something that would change the trajectory of her career.

The speaker showed a slide depicting the Conway knot and explained that mathematicians had long suspected that the knot was not, in fact, slice, but no one had been able to prove it. So what does it mean for a knot to be slice? Let's return for a moment to that four-dimensional orange. Inside of it there are disks — think of them as the surface of a plate. If a three-dimensional knot, like Conway's, can bound such a disk, then the knot is slice. If it cannot, then it is not slice.

Topologists use mathematical tools called invariants to try to determine sliceness, but for half a century, those tools had been unable to help them prove the prevailing belief that the Conway knot wasn't slice. Sitting in that lecture hall two years ago, however, Piccirillo sensed right away that the techniques she was using in a different area of topology might help these invariants better apply to the Conway knot problem. "I immediately knew that some work that I was doing for totally other reasons could at least try to answer this question," she says. She started on the problem the very next day.



The famous Conway knot, at left, and a similar one, known as the Kinoshita-Terasaka knot. By flipping the area within the red circles, one knot can be transformed into the other.Illustration by Lisa Piccirillo

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Piccirillo, who is 29, grew up in Greenwood, Maine, a town with a population of less than 900. She was an excellent student and her mom taught middle school math, but there was little in her interests to suggest that she would become a world-class mathematician.

"I was an overachiever," she says. "I rode dressage. I was very active in the youth group at my church. I did drama. I was in band. I did everything." Which is another way of saying that she wasn’t one of those math prodigies who’s programming computers and building algorithms at age 4.

When Piccirillo arrived on campus for her first year at Boston College in 2009, she was as interested in theater and other subjects as she was math. During a calculus class that year, though, she made a connection with professor J. Elisenda Grigsby. (Disclosure: I am the editor of Boston College’s alumni magazine.)

Piccirillo stood out, even if she lacked a certain polish, Grigsby recalls. “Golden-child mathematicians usually went to math camp when they were in high school and had been groomed from a young age,” she says. That wasn’t Piccirillo’s background, “but I felt a kinship to her.”

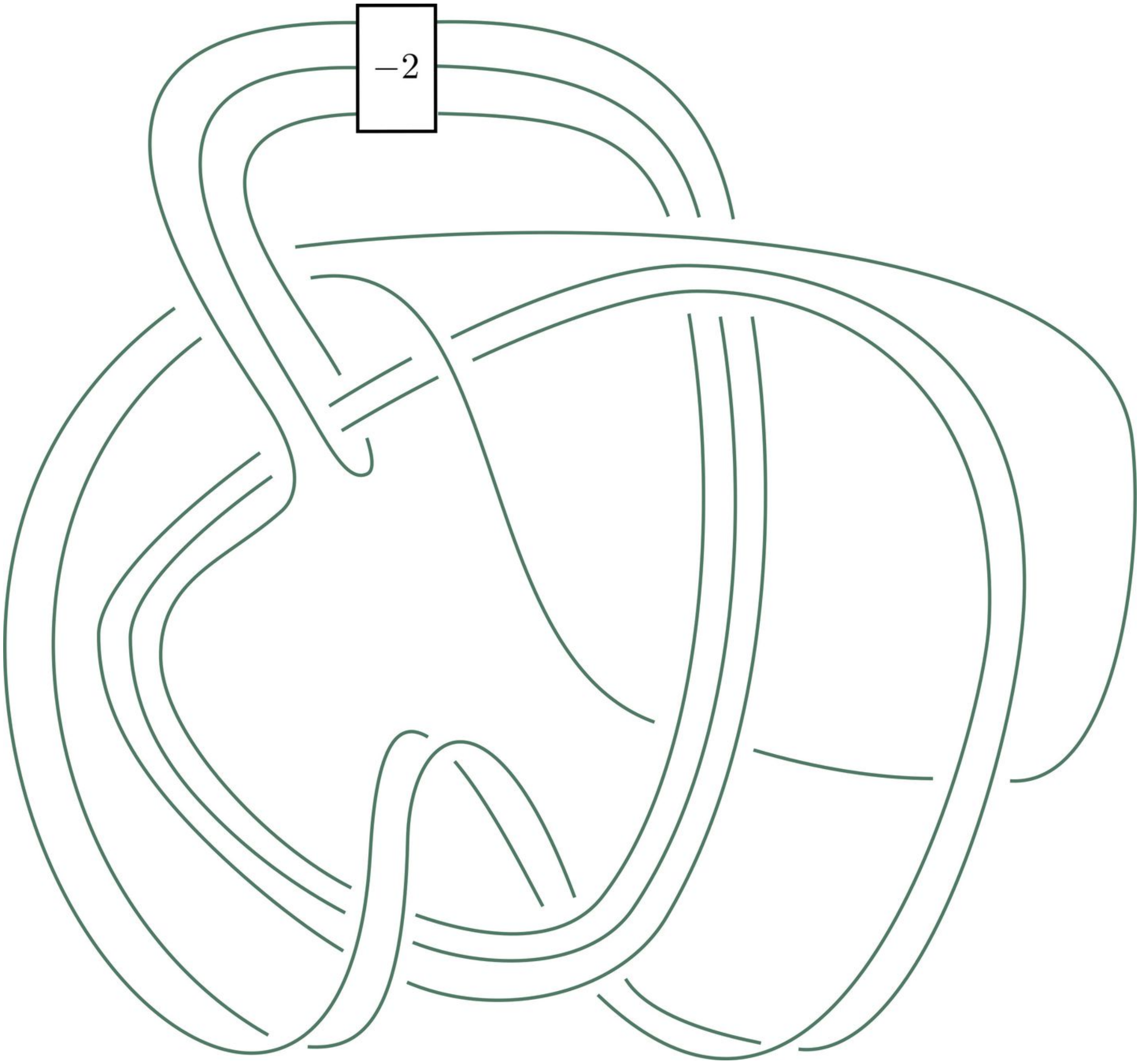
“She really encouraged me,” Piccirillo says of Grigsby. “Eli really pushed me into trying another math class, and then liking the next class. I had already started on a progression.” By her senior year, she was taking graduate-level topology courses. After graduating in 2013, she chose to pursue her doctorate at the University of Texas because of the university’s excellent topology program and its reputation as a great place for female math students. In 2014, just 28.9 percent of math and science doctorates were awarded to women, according to the National Science Foundation, but at Texas, something like 40 percent of graduate math students were women.

By and large, Piccirillo has felt welcomed and encouraged as a female mathematician. “But now and again, things happen,” she tells me. “For example, in grad school, I would receive notes in my department mailbox commenting on my appearance.”

Overall, Piccirillo excelled during her six years at the University of Texas, finding both strong mentorship and a supportive research community. The time coincided with her deepening connection to the math itself. She loved to turn problems over in her mind, thinking about how one higher-dimension shape might be manipulated to resemble an entirely different one. It was thrilling, creative work, as much about aesthetic as arriving at a particular result. “When you perform a calculation, sometimes there’s really clever tricks you can use or some ways that you can be an actual human and not a computer in the performing of the calculation,” Piccirillo says. “But when you make a logical argument — that’s entirely yours.”

Outside of her studies, Piccirillo liked to make beautiful things. She carved wooden spoons for a while, as well as large-scale woodcut prints of fish and vegetables. She and her roommate, Wiley Jennings, built a dining room table together. For a while, she was obsessed with buying and repairing ’70s Japanese motorcycles.

“She has a very, very strong sense of aesthetic,” says James Farre, a friend of Piccirillo’s from the University of Texas who specializes in geometry and is a postdoc at Yale. At Piccirillo’s level, “math that people like is often thought of and talked about as beautiful or deep.”



Piccirillo's rendering of the knot she created to solve the Conway knot problem. Illustration by Lisa Piccirillo

The day after hearing about the Conway knot problem, Piccirillo, then 27, sat down at her desk and began looking for a solution. Because much of her graduate work involved building pairs of knots that were different but shared some 4-D properties, she already knew that any two knots that share the same 4-D space also share sliceness — they're either both slice or both not slice. Since her goal was to prove that the Conway knot wasn't slice, her first step was come up with an entirely different knot with the same four-dimensional space, she explains. "Then I'll try to show that the other knot isn't slice."

She spent spare time over the next several days hand-sketching and manipulating configurations of the 4-D space occupied by the Conway knot. "I didn't allow myself to work on it during the day," she told *Quanta Magazine* earlier this year, "because I didn't consider it to be real math. I thought it was, like, my homework."

The next step was to try to prove that the knot she drew was not slice. “There are lots of tools already in the literature for doing that,” she says. She would feed the knot iterations into a computer, “and based on the data of the knot, maybe based on how its crossings look or other data that you can pull from the knot, the algorithm spits out an integer.” In less than a week, Piccirillo had created a knot that hit the sweet spot: It had the same 4-D properties as the Conway knot, and it was found by the algorithm to be not slice.

She had suddenly succeeded where countless mathematicians had failed for five decades. She had solved the Conway knot problem.

Not long after the breakthrough, Piccirillo attended a meeting with the Cameron Gordon, a University of Texas math professor. When she mentioned her solution, Gordon was skeptical. He asked Piccirillo to walk him through the steps. “Then he made me write it down, like all up on the board,” she recalls, “and then he got very excited and started yelling.”

Piccirillo submitted her solution to the *Annals of Mathematics*, and the prestigious math journal agreed to publish her paper. When I asked James Farre, the Yale postdoc, to explain the significance of having a paper published in the *Annals* he laughed for several seconds. “It’s head and shoulders the most important and influential journal in mathematics,” he says. “That’s why I’m laughing. It’s amazing and it’s so cool!”

By the time Piccirillo’s paper appeared in the journal about a year later, word of her solution had already spread throughout the math world. After graduating from UT in 2019, Piccirillo started her postdoctoral work at Brandeis. “The last time I saw her was in January,” says Wiley Jennings, her roommate in Austin, who recently completed a doctorate at Stanford. “She was out at a faculty visit here at Stanford. To be invited, as someone who has done one year or less [of postdoc study] — just finished their PhD essentially — I mean, that’s insane. It’s unheard of . . . I think that’s when I first got a hint that like, *Oh my gosh, she’s really a hotshot.*”

Postdoc positions typically run for three or four years, but Piccirillo found herself in high demand. In July, she started a new tenure-track position as an assistant professor at MIT. It’s been a whirlwind, and I wondered how her life has changed. “The practical answer is not too much,” she says. She still teaches undergrads and conducts her research. She acknowledges, though, that there sometimes is a feeling of pressure, based on what she’s already accomplished. In practice, math — for everyone — is about trying to prove simple statements and failing, basically all of the time. “So,” she says, “I’m having to relearn how to be OK with the fact that most of the time I’m failing to prove really simple stuff when I’m feeling the weight of these expectations.”

When I ask her about her goals, Piccirillo says one of her priorities is to help grow and broaden the mathematics community. “There certainly are many young women, people of color, non-heterosexual, or non-gender binary people who feel put at an arm’s length by the institution of mathematics,” she says. “It’s really important to me to help mitigate that in any small ways I can.” One important way to do that, she continues, is to help shatter the myth of the math prodigy.

When universities organize math conferences, she says, they should avoid inviting speakers who "give talks where they go really fast and they try to show you how smart they are and how hard their research is. That's not good for anyone, but it's especially not good for young people or people who are feeling maybe like they don't belong here." What those people in the audience don't know, she says, is that nobody else really understands it either.

"You don't have to be really 'smart' — whatever that means — to be a successful mathematician," Piccirillo says. "There's this idea that mathematicians are geniuses. A lot of them seem to be child prodigies that do these Olympiads. In fact, you don't have to come from that background at all to be very good at math and most mathematicians, including many of the really great ones, don't come from that sort of background."

And as Piccirillo herself proves, some of them even go on to produce work that alters the course of mathematics.

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